Pairs of Latin Squares to Counterbalance Sequential Effects and Pairing of Conditions and Stimuli

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ABSTRACT

This paper discusses methods with which one can simultaneously counterbalance immediate sequential effects and pairing of conditions and stimuli in a within-subjects design using pairs of Latin squares. Within-subjects (repeated measures) experiments are common in human factors research. The designer of such an experiment must develop a scheme to ensure that the conditions and stimuli are not confounded, or randomly order stimuli and conditions. While randomization ensures balance in the long run, it is possible that a specific random sequence may not be acceptable. An alternative to randomization is to use Latin squares. The usual Latin square design ensures that each condition appears an equal number of times in each column of the square. Latin squares have been described which have the effect of counterbalancing immediate sequential effects. The objective of this work was to extend these earlier efforts by developing procedures for designing pairs of Latin squares which ensure complete counter-balancing of immediate sequential effects for both conditions and stimuli, and also ensure that conditions and stimuli are paired in the squares an equal number of times.

INTRODUCTION

This paper discusses the methods with which one can simultaneously counterbalance immediate sequential effects and pairing of conditions and stimuli in a within-subjects design using pairs of Latin squares. Within-subjects (repeated measures) experiments are very common in human factors research. For example, if one wants to determine differences in the perceived legibility of displays, one could have each participant read texts on each display, and ask the participants to rank the displays according to legibility. Different videotext interfaces could be evaluated by having each participant use each interface to search for targets in the database.

In both these examples, the designer of the experiment must develop a scheme to ensure that the conditions and stimuli are not confounded, or the results will be uninterpretable. One commonly used scheme is to randomly order stimuli and conditions. While randomization ensures balance in the long run, it is possible that a specific random sequence may not be acceptable. An alternative to randomization is to use Latin squares (Kirk, 1982; Myers, 1979). The usual Latin square design ensures that each condition appears an equal number of times in each column and row of the square, as shown in Table 1.

Table 1. The usual Latin square with four conditions. (Letters represent conditions, columns represent order, and rows represent participants.)

A	B	C	D
B	C	D	A
C	D	A	B
D	A	B	C

METHODS FOR COUNTERBALANCING IMMEDIATE SEQUENTIAL EFFECTS

Latin squares have been described which also have the property of counterbalancing immediate sequential effects (Bradley, 1958; Williams, 1949). Bradley (1958, p. 525) states that this is an important consideration when:

> it may be suspected that reaction to any given experimental condition will be influenced (a) by the number of experimental conditions preceding it, i.e., by its ordinal position in the sequence of presentation, (b) by the particular experimental condition immediately preceding it. This situation is commonly encountered in psychological experiments when every subject is run under all experimental conditions. Such experimental designs reduce variance at the expense of introducing sequential effects such as learning, fatigue and interactions between reactions to various experimental conditions, particularly those adjacent in order of presentation.

Bradley (1958) presented a method to build a single Latin square which ensures counterbalancing of immediate sequential effects if the square has an even number of conditions. Using four conditions as an example, the steps are:

1. Start the first row by listing the first half of the letters (representing conditions) in order from left to right, skipping a location between each letter.

   A . B .

2. Complete the first row by listing the second half of the letters in order from right to left, filling the locations skipped in Step 1.

   A D B C

3. Complete the square using the letter at the top of the column as a starting point to list the remaining letters in the column in alphabetical order, wrapping from the end of the list to "A" as required, shown in Table 2.

Table 2. Latin square counterbalancing immediate sequential effects for four conditions.

A	D	B	C
B	A	C	D
C	B	D	A
D	C	A	B

4. Randomly assign the conditions to letters.

Comparing Table 2 with Table 1, it is clear that in each square each condition appears exactly once in each column. In Table 1, however, Condition A is always followed by Condition B, and is always preceded by Condition D. In Table 2, Condition A is followed exactly once by each other condition, and is preceded once by each other condition. In other words, it counterbalances immediate sequential effects. This additional control is gained at no further expense than the effort to rearrange the order of conditions in rows. The technique does not work for an odd number of conditions, however (see Table 3).

Table 3. An attempt to apply Bradley's rules to an odd number of conditions.

A	E	B	D	C
B	A	C	E	D
C	B	D	A	E
D	C	E	B	A
E	D	A	C	B

A quick examination of the conditions surrounding Condition A reveals that this attempt has failed. Condition A is followed twice by Condition E and twice by Condition C, never by Conditions B or D. Condition A is preceded twice by Conditions B and D, never by Conditions E or C. Williams (1949, p. 153) pointed out that, for an odd number of conditions, "balanced designs based on a single Latin square are impossible. It is, however, possible to achieve balance with a pair of such squares." Given the pattern in Table 3, the second square should be a mirror-image of the first. Thus, in the second square, Condition A would be followed twice by Conditions B and D, and would be preceded twice by Conditions E and C. Over the two squares, the immediate sequential effects for all conditions would again be counterbalanced (shown in Table 4). In this case, counterbalancing immediate sequential effects has a cost in that the square must be replicated. For N conditions, at least 2N participants must be run. In most situations this should not be a serious problem, since it is rare that one can achieve a reasonable level of statistical power without replicating the square.

Table 4. Counterbalancing immediate sequential effects for five conditions.

A	E	B	D	C
B	A	C	E	D
C	B	D	A	E
D	C	E	B	A
E	D	A	C	B

C	D	B	E	A
D	E	C	A	B
E	A	D	B	C
A	B	E	C	D
B	C	A	D	E

COUNTERBALANCING THE PAIRING OF CONDITIONS AND STIMULI

Things get more complicated when one tries to simultaneously counterbalance the pairing of conditions and stimuli and immediate sequential effects. Kirk (1982, p. 339-340) provides a clue to the solution:

> We have seen that a Latin square design permits an experimenter to isolate variation due to two nuisance variables while evaluating treatment effects. A Graeco-Latin square design . . . permits the isolation of three variables. (It) consists of two superimposed orthogonal Latin squares. . . . Two Latin squares are orthogonal if and only if when they are superimposed, each letter of one square occurs once and only once with each letter of the other square.

To counterbalance the pairing of conditions and stimuli, orthogonal Latin squares must be developed for conditions and stimuli, which are then superimposed. The solutions for even and odd numbers of conditions/stimulus sets are slightly different.

Solution for an odd number

Using the procedure described by Bradley (1958), build a pair of Latin squares which counterbalance immediate sequential effects (see Table 4). Copy the squares using numbers to represent stimulus sets, as shown in Table 5. Then superimpose the squares diagonally to produce the squares in Table 6.

Table 5. Developing orthogonal Latin squares for five conditions and stimulus sets. (Letters represent conditions, numbers represent stimulus sets.)

A	E	B	D	C	1	5	2	4	3
B	A	C	E	D	2	1	3	5	4
C	B	D	A	E	3	2	4	1	5
D	C	E	B	A	4	3	5	2	1
E	D	A	C	B	5	4	1	3	2

C	D	B	E	A	3	4	2	5	1
D	E	C	A	B	4	5	3	1	2
E	A	D	B	C	5	1	4	2	3
A	B	E	C	D	1	2	5	3	4
B	C	A	D	E	2	3	1	4	5

Table 6. Counterbalancing immediate sequential effects and condition/stimulus pairing for five conditions. (Letters represent conditions, numbers represent stimulus sets.)

A3	E4	B2	D5	C1
B4	A5	C3	E1	D2
C5	B1	D4	A2	E3
D1	C2	E5	B3	A4
E2	D3	A1	C4	B5

C1	D5	B2	E4	A3
D2	E1	C3	A5	B4
E3	A2	D4	B1	C5
A4	B3	E5	C2	D1
B5	C4	A1	D3	E2

Since the condition squares were developed using Bradley's rules, they are counterbalanced for immediate sequential effects. Since the stimulus squares were copied from the condition squares, they must be counterbalanced for immediate sequential effects. When the squares are superimposed, each condition is paired with the stimuli equally. Therefore, this pair of squares counterbalances both immediate sequential effects and the pairing of conditions and stimuli.

Solution for an even number

The solution for an odd number of conditions/stimulus sets was relatively easy since taking the mirror-image of the first square produced an orthogonal square. Unfortunately, taking the mirror-image of an even square just reproduces the square (see Table 2). To generate the second square for an even number of conditions, swap each pair of columns, as shown in Table 7.

Table 7. Generating the second square for four conditions.

A	D	B	C
B	A	C	D
C	B	D	A
D	C	A	B
D	A	C	B
A	B	D	C
B	C	A	D
C	D	B	A

While it is different from the first square, the second square is also counterbalanced for immediate sequential effects. In each square, each condition is preceded by and followed by each other condition exactly once. Next, copy the squares using numbers to represent stimulus sets (shown in Table 8).

Table 8. Generating the squares for the stimulus sets for four conditions.

A	D	B	C	1	4	2	3
B	A	C	D	2	1	3	4
C	B	D	A	3	2	4	1
D	C	A	B	4	3	1	2
D	A	C	B	4	1	3	2
A	B	D	C	1	2	4	3
B	C	A	D	2	3	1	4
C	D	B	A	3	4	2	1

If the squares are superimposed diagonally with their current structure, conditions and stimuli will not be properly paired. For example, over both squares, Stimulus set 1 would be paired only with Conditions B and D. Rotating the rows of the second square of numbers by one position before superimposition will correct the problem (see Table 9).

Table 9. Adjusting the second square of numbers.

A	D	B	C	1	4	2	3
B	A	C	D	2	1	3	4
C	B	D	A	3	2	4	1
D	C	A	B	4	3	1	2
D	A	C	B	1	2	4	3
A	B	D	C	2	3	1	4
B	C	A	D	3	4	2	1
C	D	B	A	4	1	3	2

This adjustment does not affect the counterbalancing of immediate sequential effects since columns (order of presentation) remain counterbalanced when rows (participants) are reordered. Now the squares can be superimposed, as shown in Table 10.

In the first square of Table 10, Condition A is paired with Stimulus sets 1 and 3 twice, and in the second square, with Stimulus sets 2 and 4 twice. Strictly speaking, these are not Graeco-Latin squares since conditions and stimulus sets are not equally paired within squares. However, the cumulative effect over pairs of squares is complete condition/stimulus set counterbalancing with simultaneous counterbalancing of immediate sequential effects.

Table 10. Counterbalancing immediate sequential effects and condition/stimulus pairing for four conditions. (Letters represent conditions, numbers represent stimulus sets.)

A1	D2	B4	C3
B2	A3	C1	D4
C3	B4	D2	A1
D4	C1	A3	B2
D1	A4	C2	B3
A2	B1	D3	C4
B3	C2	A4	D1
C4	D3	B1	A2

ANALYSIS OF THE DESIGNS

A comprehensive treatment of analysis of these designs is outside the scope of this paper. Three candidates are repeated measures analysis of variance (most common) (Kirk, 1982; Myers, 1979), multivariate analysis of variance (viewing each condition's measure as one of a set of correlated dependent measures) (Cliff, 1987), and the Friedman two-way layout test (the nonparametric version of the repeated measures analysis of variance) (Hollander and Wolfe, 1973). Each method has its advantages and drawbacks, largely dependent upon the nature of the data which has been collected.

CONCLUSIONS

I do not have a formal proof that the procedure will work for any number of conditions, but I have built the squares from two to ten conditions/stimulus sets, and they work. (See Appendix A.) The procedures and squares described allow a human factors engineer to design an efficient, well-counterbalanced study without relying on chance. The technique can be applied to any experiment in which different stimulus sets must be efficiently paired with conditions in a within-subjects design.

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APPENDIX A. Pairs of Latin Squares to Counterbalance Sequential Effects and Pairing of Conditions and Stimuli for 2 to 8 Conditions/Stimulus Sets

Letters represent experimental conditions.
Numbers represent stimulus sets.
Columns represent the order of presentation.
Rows represent participants.

2 Conditions/Stimulus Sets

A1	B2
B2	A1

B1	A2
A2	B1

3 Conditions/Stimulus Sets

A2	C3	B1
B3	A1	C2
C1	B2	A3

B1	C3	A2
C2	A1	B3
A3	B2	C1

4 Conditions/Stimulus Sets

A1	D2	B4	C3
B2	A3	C1	D4
C3	B4	D2	A1
D4	C1	A3	B2

D1	A4	C2	B3
A2	B1	D3	C4
B3	C2	A4	D1
C4	D3	B1	A2

5 Conditions/Stimulus Sets

A3	E4	B2	D5	C1
B4	A5	C3	E1	D2
C5	B1	D4	A2	E3
D1	C2	E5	B3	A4
E2	D3	A1	C4	B5

C1	D5	B2	E4	A3
D2	E1	C3	A5	B4
E3	A2	D4	B1	C5
A4	B3	E5	C2	D1
B5	C4	A1	D3	E2

6 Conditions/Stimulus Sets

A1	F2	B6	E3	C5	D4
B2	A3	C1	F4	D6	E5
C3	B4	D2	A5	E1	F6
D4	C5	E3	B6	F2	A1
E5	D6	F4	C1	A3	B2
F6	E1	A5	D2	B4	C3

F1	A6	E2	B5	D3	C4
A2	B1	F3	C6	E4	D5
B3	C2	A4	D1	F5	E6
C4	D3	B5	E2	A6	F1
D5	E4	C6	F3	B1	A2
E6	F5	D1	A4	C2	B3

7 Conditions/Stimulus Sets

A4	G5	B3	F6	C2	E7	D1
B5	A6	C4	G7	D3	F1	E2
C6	B7	D5	A1	E4	G2	F3
D7	C1	E6	B2	F5	A3	G4
E1	D2	F7	C3	G6	B4	A5
F2	E3	G1	D4	A7	C5	B6
G3	F4	A2	E5	B1	D6	C7

D1	E7	C2	F6	B3	G5	A4
E2	F1	D3	G7	C4	A6	B5
F3	G2	E4	A1	D5	B7	C6
G4	A3	F5	B2	E6	C1	D7
A5	B4	G6	C3	F7	D2	E1
B6	C5	A7	D4	G1	E3	F2
C7	D6	B1	E5	A2	F4	G3

Corrected

8 Conditions/Stimulus Sets

A1	H2	B8	G3	C7	F4	D6	E5
B2	A3	C1	H4	D8	G5	E7	F6
C3	B4	D2	A5	E1	H6	F8	G7
D4	C5	E3	B6	F2	A7	G1	H8
E5	D6	F4	C7	G3	B8	H2	A1
F6	E7	G5	D8	H4	C1	A3	B2
G7	F8	H6	E1	A5	D2	B4	C3
H8	G1	A7	F2	B6	E3	C5	D4

H1	A8	G2	B7	F3	C6	E4	D5
A2	B1	H3	C8	G4	D7	F5	E6
B3	C2	A4	D1	H5	E8	G6	F7
C4	D3	B5	E2	A6	F1	H7	G8
D5	E4	C6	F3	B7	G2	A8	H1
E6	F5	D7	G4	C8	H3	B1	A2
F7	G6	E8	H5	D1	A4	C2	B3
G8	H7	F1	A6	E2	B5	D3	C4